Causal Graphs and Structurally Restricted Planning

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Introduction

- Well-acknowledged: although planning in general NP-hard, real-world planning problems exhibit *structure*
- Exploiting such structure (either implicitly or explicitly) has been focus of much research
- Feature of planning instances that has attracted attention: *causal graph* – directed graph whose edges describe variable dependencies (Knoblock '94)
- Examples:
    - used as basis for heuristics in Fast Downward (Helmert 2006)
    - factoring planning problems (Knoblock '94, Brafman & Domshlak '06)

The causal graph

- Def: The *causal graph* of a planning instance Π is the directed graph where:
  - the vertices are the variables of Π,
  - there is an edge (u, v) if $u \neq v$ and there exists an operator contaning v in the postcondition and u somewhere (in either pre- or postcondition)
- Complexity results based on causal graphs (sampling):
  - Poly-time algorithms for problems defined by restricting causal graph (Jonsson and Bäckström '98), (Brafman and Domshlak '03), (Jonsson '07)
  - Complexity of certain families of causal graphs (Domshlak and Dinitz '01), (Helmert '04), (Gimenez and Jonsson '08)
  - Complexity results for sequentially-optimal planning (Katz and Domshlak '07)
  - Planning with bounded local depth (Brafman and Domshlak '06)

A complete classification?

- Concrete families of causal graphs have been studied
- Let $\mathcal{C}$ be a set of directed graphs, and define Planning($\mathcal{C}$) to be the planning problem restricted to instances having causal graph in $\mathcal{C}$
- Examples:
  - "Fork" graphs (Domshlak and Dinitz '01)
  - Directed paths (Gimenez and Jonsson '08)
- Can a complete classification be given? i.e., a systematic description of the sets of graphs $\mathcal{C}$ describing each problem Planning($\mathcal{C}$) as tractable / intractable
- This paper: yes

Classification theorem

- For an undir. graph G , define cc-size(G) to be be the size of the largest *connected component* –
set of vertices S such that any two $u, v \in S$ have path to each other
- For a dir. graph C, define cc-size(C) to be equal to cc-size(G) where G is the undir. graph obtained by "forgetting" orientations of C
- Example: consider the dir. graph C

$$u \longrightarrow v \longrightarrow w \longleftarrow x$$

the induced undir. graph G is

$$u - v - w - x$$

and cc-size(C) = cc-size(G) = 4

Classification theorem

- For a set of directed graphs $\mathcal{C}$, let us say that cc-size($\mathcal{C}$) is *bounded* if there exists a constant k such that: for all $C \in \mathcal{C}$, it holds that cc-size(C) $\leq k$
- Thm: Let $\mathcal{C}$ be a set of directed graphs.
  - If cc-size($\mathcal{C}$) is bounded, then PlanGen($\mathcal{C}$) is poly-time solvable;
  - Otherwise, PlanExist($\mathcal{C}$) is not poly-time decidable (assuming W[1] $\not\subseteq$ nu-FPT)
- Note: no size restriction on variable domains (other than finiteness)
- Tells us which tractable classes can be explained to be tractable solely via the causal graph
- It is possible, however, that graph restrictions can be combined with other features to give tract. results...

Reversible Planning

- Our second theorem studies the feature of *reversibility*
- Say that a planning inst. Π is *reversible* if for any state s reachable from the initial state,
  the initial state can be reached from s
- Feature present in many benchmark domains
- Thm: Let $\mathcal{C}$ be a set of directed graphs.
  - If scc-size($\mathcal{C}$) is bounded, then RevPlanGen($\mathcal{C}$) is poly-time solvable;
  - Otherwise, RevPlanExist($\mathcal{C}$) is not poly-time decidable (assuming W[1] $\not\subseteq$ nu-FPT)
- Here, the relevant parameter of a dir. graph C is scc-size(C), size of the largest strongly connected component of C

Why parameterized complexity?

- Our hardness results are proved relative to assumption $W[1] \not\subseteq$ nu-FPT – from *parameterized complexity*
- You may ask: why do we not prove them relative to (better known) assumption $P \neq NP$?
- Answer: this is not possible!
  The problem family PlanExist($\mathcal{C}$) is too rich
- Thm (follows from Bodirsky & Grohe '08):
  There exists a set of dir. graphs $\mathcal{C}$ such that PlanExist($\mathcal{C}$) is "NP-intermediate", i.e. not in P nor NP-hard
  (assuming $P \neq NP$)
- Intuition: take $\mathcal{C}$ to be a sparse set, such as

$$\{ \text{ cliques of size } 2^{2^n} \}$$

What is parameterized complexity?

- Parameterized complexity: a “two-dimensional” complexity theory
- Considers problems that have two parts: main part and parameter
- Example - CLIQUE:
  Given a graph G, does it contain a clique of size k?
- Observe: for each *fixed* k, CLIQUE can be solved in time $O(n^k)$
- Each “slice” is poly-time tractable, but we have a bad dependence on k
- One may ask if there are algorithms without bad dependence, i.e., is there a constant c such that each slice k of CLIQUE can be solved in time $O(n^c)$?
- Conjectured answer *no*, and this is equivalent to W[1] $\not\subseteq$ nu-FPT

How we use parameterized complexity

- Our hardness results are achieved by reduction from CLIQUE
- In order to employ assumption, reductions have to avoid “bad dependence” on k

Proofs

- Tractability of PlanGen($\mathcal{C}$) when cc-size($\mathcal{C}$) is bounded:
  - instances consist of disjoint components that do not interact
  - compute the entire transition graph for each component
- Intractability of PlanExist($\mathcal{C}$) for unbounded cc-size($\mathcal{C}$):
  - Two cases: bounded pre-degree and unbounded pre-degree
  - Pre-degree of a vertex v in a graph is

$$|\{u \ : \ u \text{ has a dir. path to } v\}|$$

  - Say a set of graphs $\mathcal{C}$ has unbounded pre-degree if the pre-degree of vertices is unbounded, i.e.,
    for any $m > 1$, we can find a graph $C \in \mathcal{C}$ having a vert. v of pre-degree $\geq m$

Intractability - unbounded pre-degree

- Example: set of directed paths $\{\rightarrow, \rightarrow\rightarrow, \rightarrow\rightarrow\rightarrow, \ldots\}$
  Has unbounded pre-degree
- How to prove hardness for unbounded pre-degree?
- Let (G, k) be an instance of CLIQUE
- Let $C \in \mathcal{C}$ be a graph having a vertex d of pre-degree $\geq k$
- Let $v_1, \ldots, v_k$ be vertices having paths to d
- Idea: create planning inst. where each var. v_i can "commit" to a vertex in G
- Commitments can be "announced" via messages to d along paths
- Var. d can reach goal state iff the committed-to vertices form a clique in G

Intractability - bounded pre-degree

- Example & representative case: set of "zig-zags"
  $\{\rightarrow\leftarrow, \rightarrow\leftarrow\rightarrow\leftarrow, \rightarrow\leftarrow\rightarrow\leftarrow\rightarrow\leftarrow, \ldots\}$
  Has bounded pre-degree
- How to prove hardness for bounded pre-degree?
- Use notation $a_1 \rightarrow b_1 \leftarrow a_2 \rightarrow b_2 \leftarrow a_3 \rightarrow \ldots$
- Given an instance (G, k) of CLIQUE …
- The vars. a_i each "broadcast" a sequence of vertices $w_1, \ldots, w_k \in G$
- The goal state can be reached iff all broadcast the same clique in G
- Each var. b_i checks one edge $\{w_i, w_j\}$
- The vars. b_i also check that all a_j broadcast the same vertices

Interaction networks

- We begin study of an object which we call the *interaction network*
- It is what one gets by "removing" values from operators in a planning inst.
- The interaction network is "finer"/"contains more information" than the causal graph:
  from the int. network, we can compute the causal graph, but not vice-versa
- Def: the interaction network of an inst. Π is the pair (V, E) where V is the vars. of Π and

  $$E = \{(\mathrm{VarsPre}(a), \mathrm{VarsPost}(a)) \mid a \text{ is an operator of } \Pi\}$$

  where $\mathrm{VarsPre}(a)$ and $\mathrm{VarsPost}(a)$ denote the variables of the pre- and post-conditions of an operator a

Interaction networks

- Thm: the planning instances having interaction networks of the form $\{(\{v_1\},\{v_2\}),(\{v_2\},\{v_3\}),(\{v_3\},\{v_4\}),\ldots\}$ are poly-time tractable
- Note: this tractability result cannot be described using causal graphs!

  The instances shown to be tractable have directed path causal graphs
- Bylander's ('94) result on the poly-time tractability of precondition-free planning can also be described in terms of int. networks

Interaction networks

- Our inspiration to study the int. network comes (by analogy) from the work on structurally restricted constraint satisfaction (Gottlob et al. '00), (Grohe '07), ...
- Research question: give a complete complexity classification of all sets of int. networks
- We look forward to future research along these lines!