Bounded Rationality, Strategy Simplification, and Equilibrium

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Bounded Rationality

- Frequently raised criticism of game theory: predictions clash with empirical observations, due to assumption: agents have unbd. computational power
- Criticism has motivated study of models of *bounded rationality* (Simon '69)
- One model: *machine game*
  - Players choose finite-state automata that represent strategies for repeated games (Neyman '85, Rubinstein '86, Kalai & Stanford '88, ...)
- Finite-state automata can be viewed as formalization of players having bounded-size memory
  - well-studied in computer science

Nash equilibrium a la Rubinstein

- Rubinstein with Abreu and Piccione ('86, 88, '93) studied forms of Nash equilibrium where strategy complexity is taken into account
- Studied maximally simplified strategies, where a player's strategy cannot be simplified without reducing his/her payoff
  - Studied machine game
  - Strategy complexity $\equiv$ number of states (memory size)
- Basic supposition: in addition to maximizing payoff, players desire to minimize strategy complexity

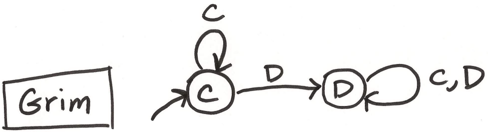
Nash equilibrium a la Rubinstein

- Why is strategy simplicity desirable?
- (Osborne & Rubinstein) suggest complex strategies may be
  - more expensive to execute,
  - more likely to break down,
  - harder to learn, &
  - costly to maintain.
- Following (Rubinstein '86), it can be suggested that maximally simplified strategies capture phenomena observed in real life:
  - Institutions, organizations, and human abilities may degenerate or be reduced if they contain unnecessary or redundant components

A new notion of equilibrium

- We introduce/study a new notion of equilibrium that captures maximally simplified strategies
  - ...but with respect to a more careful, conservative simplification procedure
- Motivation: in Rubinstein model, a player simplifies without considering whether or not simplification may incent other players to deviate
  - this "liberal" mode of simplification may spoil desirable outcomes, e.g. outcomes that are Nash equilibr. in usual payoff sense
- Let's consider an example: infinitely repeated Prisoner's Dilemma...

Example: grim trigger



- (Grim, Grim) not an equilibrium in Rubinstein model
- Prisoner's Dilemma:

	C	D
C	(2,2)	(-1,3)
D	(3,-1)	(0,0)

A new notion of equilibrium

- In Rubinstein model: a player simplifies his strategy as long as he can maintain his payoff
- In our model: players are more forward-looking, and simplify if (in addition) no other player can profitably deviate post-simplification
- That is: in considering simplifications, players are averse to potential payoff-motivated deviations by other players
- In other words: players only simplify if the result is a NE
- Our notion: *lean equilibrium* - an outcome of strategies at NE such that no player can both individually simplify *and* preserve the property of being at NE

A new notion of equilibrium: example

Grim trigger strategy paired with itself, *(Grim, Grim)*, *is* a lean equilibrium (in ∞-repeated Prisoner's dilemma)

- *(Grim, Grim)* is a NE
- *Grim* strategy has two states
- Consider *(S, Grim)* where S is a simplification. We claim not a NE.
  - S must have one state, and must always cooperate (to be best response to *Grim*)
  - But, this is not a NE! Other player can then profitably deviate by always defecting.

Summary of technical results

Results concern the machine game.

1. We give techniques for establishing that outcomes are at lean equilibrium, and illustrate their use with examples
2. We present structure of equilibria results.
   For a "number-of-transitions" complexity measure, we give a precise description of equilibria structure.
   This description shows that the machine structure can be inferred from a third-party observer that only views the sequence of produced actions!

Definitions

- A *strategic game* is a tuple $(N, (A_i), (\preceq_i))$ where
  - $N = \{1, \ldots, n\}$ is set of players
  - A_i is set of actions for player i
  - $\preceq_i$ is a preference relation on $\times_{j \in N} A_j$ for player i
- A Nash equilibr. is a profile a^* such that

$$(a^*_{-i}, a_i) \preceq_i (a^*_{-i}, a^*_i) \text{ (for all } i)$$

- We study games where each player has a *complexity order* $\trianglelefteq_i$ – a binary relation on A_i
  Intended use: $b_i \trianglelefteq_i a_i$ if b_i has same/lower complexity than a_i
- A *lean equilibr.* is a Nash equilibr. a^* such that
  if $a_i \lhd a^*_i$ then (a^*_{-i}, a_i) is not a Nash eq.

Existence of lean equilibria

- Prop: Suppose that G a strategic game with complexity orders $(\trianglelefteq_i)$ that are partial orders that are *well-founded*:

  for all $i \in N$, $a_i \in A_i$, there exists a bound on the length of a chain $c_1 \triangleleft_i \cdots \triangleleft_i c_K \triangleleft_i a_i$

  Then for every Nash eq. a^* of G, there exists a lean eq. $b \in A$ with $b_i \trianglelefteq a_i^*$ (for all i).
- Note: the complexity orders we study are total orders and are well-founded

Abreu-Rubinstein equilibria

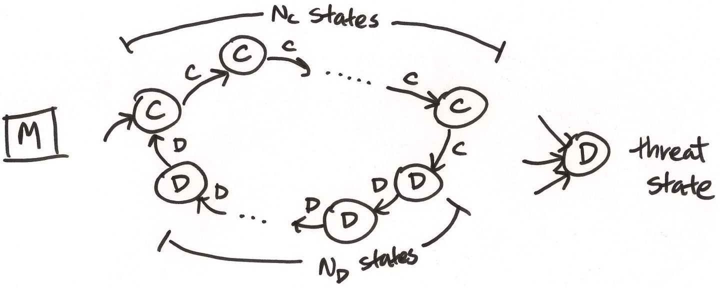
- An equilibrium notion studied by Abreu & Rubinstein ('88).
- Idea: each player wants to maximize payoff, but also prefers simpler strategies if they sustain payoff
- Def: Let G be a strategic game with complexity orders $(\trianglelefteq_i)$. A profile a^* is an Abreu-Rubinstein equilibrium if:
  1. $(a^*_{-i}, a_i) \preceq_i (a^*_{-i}, a^*_i)$, and
  2. $(a^*_{-i}, a^*_i) \preceq_i (a^*_{-i}, a_i)$ implies that $a_i \triangleleft_i a^*_i$ does not hold.
- Prop: Let G be a strategic game with complexity orders $(\trianglelefteq_i)$. Every Abreu-Rubinstein equilibrium is a lean equilibrium.

Machines and Complexity Measures

- A machine (succinctly) defines a strategy in an infinitely repeated game (limit of means used to compute payoff)
- A pair of machines naturally induces a sequence of action pairs (G-outcomes)
- We study three complexity measures. Let M_i be a machine.
  1. $|Q_i|$ - the number of states
  2. $|R_i|$ - the number of *normal* states.
     A *threat* state $\approx$ a state that forces the other player to his minmax payoff. All other states called *normal* states.
  3. $||\delta_i||$ - the number of *normal transitions*:

     $$||\delta_i|| = |\{(q_i, s_j) \in R_i \times S_j \mid \delta_i(q_i, s_j) \in R_i\}|$$
- Each measure induces a complexity order, e.g. for $|Q_i|$, have: $M_i \trianglelefteq_i M_i'$ if and only if $|Q_i| \leq |Q_i'|$

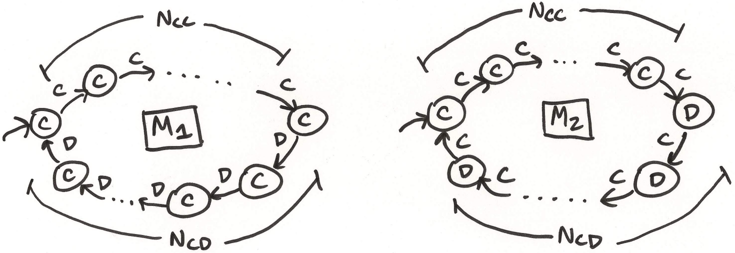
Example



- $N_c, N_D \geq 1$.
- $|R| = \|\delta\| = N_c + N_D$, $|Q| = N_c + N_D + 1$.
- (M, M) Abreu-Rubinstein eq. WRT $|R|$, $\|\delta\|$
  - Can be proved that any best response has $|R| \geq N_c + N_D$
- (M, M) NOT Abreu-Rubinstein eq. WRT $|Q|$

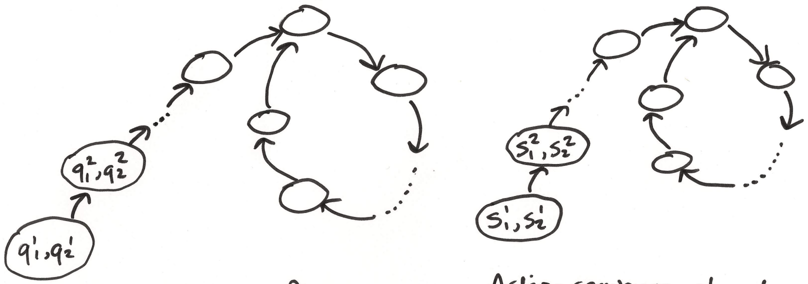
(M, M) is lean eq. WRT $|Q|$.

Example



- Threat states not drawn
- Assume $N_{CC} > 0$; N_{CC}, N_{CD} relatively prime
- (M_1, M_2) not Abreu-Rubinstein eq. WRT $|R|, \|\delta\|, |Q|$
  - Player 1 can simplify + preserve payoff : 
- (M_1, M_2) is lean eq. WRT $|R|, \|\delta\|, |Q|$

Structure Theorem



State sequence forms a ρ ⇒ Action sequence also does

Thm. Suppose (M_1, M_2) a lean eq. of G_m WRT $\|\delta\|$ having a strictly enforc. payoff profile. Then the two "ρ"s shown have the same structure, and this is the structure of M_1, M_2.

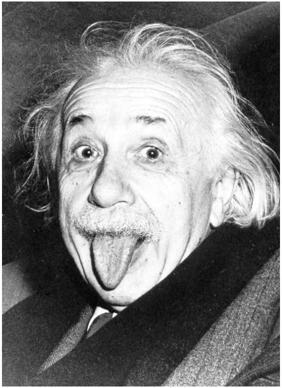
Structure Theorem: formalization

- A pair of machines (M_1, M_2) naturally induces
  - a sequence $(q^t)_{t \geq 1}$ of state pairs, and
  - a sequence $(s^t)_{t \geq 1}$ of action pairs with $s^t = (\lambda_1(q_1^t), \lambda_2(q_2^t))$
- We define equivalence relations on the natural numbers:
  - $t \equiv_s t'$ iff for all $n \geq 0$, $s^{t+n} = s^{t'+n}$
  - $t \equiv_q t'$ iff for all $n \geq 0$, $q^{t+n} = q^{t'+n}$ (iff $q^t = q^{t'}$)
  - $t \equiv_i t'$ iff $q_i^t = q_i^{t'}$ (for $i \in \{1, 2\}$)
- Some basic relationships:
  - if $t \equiv_q t'$, then $t \equiv_s t'$
  - $t \equiv_q t'$ if and only if $t \equiv_1 t'$ and $t \equiv_2 t'$
- Thm: If (M_1, M_2) a lean eq. wrt $||\delta||$ having a strictly enforceable payoff profile, then the equivalence relations $\equiv_s$, $\equiv_q$, $\equiv_1$, $\equiv_2$ are all equal.

Discussion/Wrap-up

- We introduced a notion of equilibrium for games in which there is a notion of *strategy complexity* on the players' action sets
- As with the equilibrium studied by Abreu & Rubinstein, captures "maximally simplified strategies".
  Intuition:
  - Abreu-Rubinstein equilibrium - player simplifies if can maintain payoff
  - Lean equilibrium - player simplifies if can maintain Nash equilibrium
- Broad research direction: notions of equilibria where simplified strategies preferred by players, but simplicity not tied directly into payoff

As Einstein said...



"Everything should be made as simple as possible, but not simpler."

Discussion/Wrap-up

- Not studied here: computational complexity of deciding if a player can simplify without incenting others to deviate
- The complexity of such *meta-reasoning* is potentially relevant
  - Interesting issue for future work
- There may be scenarios where meta-reasoning may be "inexpensive" compared to simplification
  - Example: reasoning about how to reconfigure a large organization may be cheaper than actually reconfiguring it

Discussion/Wrap-up

- Studied lean equilibria in machine games
- One suggestion for future work: study lean equilibria in other games where there is (or one can define) a notion of strategy complexity